

# SOME THOUGHTS

conventional and unconventional

ON EQUILIBRATION

" THERMAL-FEST "

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# 1. THERMAL FITS $\leftrightarrow$ THERMALIZATION ?

$E_T$  - spectra, relative abundances of emitted hadrons in  $A+A$  (even  $p+p/\bar{p}$ ,  $e^+e^-$  ?) collisions are often well described by thermal fits:

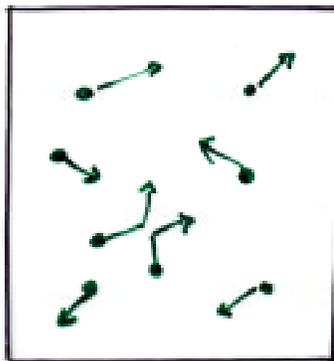
$$(*) \quad n_i(E_T) = \left( e^{\beta_i E_T (v_i) - \beta_i \mu_i} \pm 1 \right)^{-1}$$

$\beta_i, v_i, \mu_i$  are related to freeze-out conditions

$$\lambda_f^{(i)} \approx R_{\text{system}}$$

$$\text{or } \tau_r^{(i)} \approx \tau_{\text{system}} .$$

But is (\*) indicative of thermalization  $\equiv$  attainment of thermal equilibrium ?



Does this system equilibrate ?

Does an isolated system equilibrate ?

## 2. CANONICAL DISTRIBUTIONS & THERMAL FLUCTUATIONS

### FROM (CHAOTIC) QUANTUM DYNAMICS

Historically, thermal equilibrium is defined as the Gibbs ensemble

$$\hat{\rho}_{th} = Z^{-1} \exp \{ -(\hat{H} - \mu \hat{N}) / T \}.$$

So, what about a single event?

Consider an isolated quantum system of many particles ( $N$ ) in an energy eigenstate.

For a chaotic quantum system, these states satisfy Berry's conjecture

$$\psi_{\alpha}(\vec{x}) = \int d\vec{p} A_{\alpha}(\vec{p}) \delta(\vec{p}^2 - 2mE_{\alpha}) e^{i/\hbar \vec{p} \cdot \vec{x}}$$

where  $A(\vec{p})$  is a Gaussian random variable

$$\langle A_{\alpha}^*(\vec{p}) A_{\beta}(\vec{p}') \rangle = \delta_{\alpha\beta} \delta^{3N}(\vec{p} - \vec{p}').$$

Take an initial state

$$\phi_0(\vec{x}, t=0) = \sum_{\alpha} C_{\alpha} \psi_{\alpha}(\vec{x})$$

and look at its evolution.

M. Srednicki

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Assume

$$\Delta \equiv \left[ \sum_{\alpha} |C_{\alpha}|^2 (E_{\alpha} - \bar{E})^2 \right]^{1/2} \ll \bar{E} = \sum_{\alpha} |C_{\alpha}|^2 E_{\alpha}$$

$\psi_0(\vec{x})$  evolves as

$$\psi_0(\vec{x}, t) = \sum_{\alpha} C_{\alpha} e^{-i/\hbar E_{\alpha} t} \psi_{\alpha}(\vec{x})$$

Consider momentum distribution of a single particle:

$$\begin{aligned} f_1(p, t) &= \int dp_2 \dots dp_N |\tilde{\Phi}_0(\vec{p}, t)|^2 \\ &= \sum_{\alpha\beta} C_{\alpha}^* C_{\beta} e^{i/\hbar (E_{\alpha} - E_{\beta}) t} \Phi_{\alpha\beta}(p) \end{aligned}$$

with

$$\Phi_{\alpha\beta}(p) = \int dp_2 \dots dp_N \tilde{\psi}_{\alpha}^*(\vec{p}) \tilde{\psi}_{\beta}(\vec{p})$$

For  $N \rightarrow \infty$ :

$$\langle \Phi_{\alpha\alpha}(p) \rangle = \frac{e^{-p^2/2mT_{\alpha}}}{(2\pi m T_{\alpha})^{3/2}} \equiv f_{th}(p, T_{\alpha}) \quad \text{with } E_{\alpha} = \frac{3}{2} N T_{\alpha}$$

with fluctuations  $O(e^{-\frac{3}{4}N})$  !

For a general initial state

$$\langle f_1(p, t) \rangle = f_{th}(p, \bar{T}) [1 + O(\Delta/\bar{E})]$$

$$\text{and } \bar{E} = \frac{3}{2} N \bar{T}.$$

Fluctuations w.r.t. initial state  $\phi_0 \leftrightarrow C_\alpha$  are decaying at least as  $t^{-1}$ .

"Eigenstate thermalization"

Can be easily generalized to Bose/Fermi statistics.

For more general observables

Srednicki  
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$$\langle A(t) \rangle = \sum_{\alpha\beta} C_\alpha^* C_\beta e^{i\frac{t}{\hbar}(E_\alpha - E_\beta)} A_{\alpha\beta}.$$

Time average

$$\bar{A} \equiv \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau dt \langle A(t) \rangle = \sum_{\alpha} C_{\alpha\alpha} A_{\alpha\alpha}$$

can be shown to depend only on  $\bar{E}$  for large  $N$ :

$$\bar{A} = A(\bar{E}) + O(\Delta/\bar{E})^2 \quad (N \rightarrow \infty)$$

where  $A(\bar{E})$  is the microcanonical average of  $A_{\alpha\alpha}$  for  $E_\alpha \approx \bar{E}$ .

Moreover:

$$\overline{(\langle A(t) \rangle - \bar{A})^2} \rightarrow 0 \quad \text{for } N \rightarrow \infty.$$

For thermal fluctuations, look at

$$\overline{A^2} \equiv \lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau dt \langle A^2(t) \rangle = \sum_{\alpha} |C_{\alpha}|^2 (A^2)_{\alpha\alpha}$$

$$\hookrightarrow A^2(\overline{E}) \quad \text{for } N \rightarrow \infty.$$

Thus, the long-time average of the quantum fluctuations is given by the thermal fluctuation of  $A$ :

$$\overline{\langle A^2(t) \rangle} - \langle A(t) \rangle^2 = \langle (A - \overline{A})^2 \rangle_{th}.$$

When does this picture apply?

Two general conditions for quantum chaos to exist:

$$(1) \quad \overline{\Delta} \ll \hbar \gamma_{slow}$$

$\overline{\Delta} \equiv$  mean energy spacing of neighb. states

$\gamma_{slow} \equiv$  slowest relevant classical rate

$$(2) \quad \overline{E} \gg \hbar \gamma_{fast}$$

$\overline{E} \equiv$  total system energy

$\gamma_{fast} \equiv$  fastest relevant classical rate

Srednicki, cond-mat/9605127

Agam, Andreev, Simons, Altshuler, PRL 76 (96) 3947

$\gamma_{\text{slow}}$  is given by diffusive processes, i.e.

by the Thouless rate:  $\frac{D}{L^2} = \frac{l_f v}{L^2}$ .

For  $\bar{E}/N$  fixed,  $N \sim L^3$ , one has  $\gamma_{\text{slow}} \sim N^{-2/3}$ .

But  $\bar{\Delta} \sim \bar{E} e^{-S(\bar{E})} \sim N e^{-Ns}$ , and thus condition (i) is satisfied for large  $N$ .

(General theory of  $\gamma_{\text{slow}}$  is based on eigenvalues of the Perron-Frobenius operator.)

$\gamma_{\text{fast}}$  is generally the (mean free time)<sup>-1</sup>:  $\frac{v}{l_f} N$

More generally:  $\gamma_{\text{fast}} = h_{ks} \equiv \sum_{\alpha} \lambda_{\alpha}^+$

For Yang-Mills fields ( $SU(2)$ )  $h_{ks} \approx \frac{1}{10} g^2 \bar{E}$

and thus

$$\hbar \gamma_{\text{fast}} \approx \frac{4\pi}{10} \alpha_s \bar{E} \ll \bar{E} \quad (\alpha_s \ll 1)$$

**Conclusion:**

Eigenstate thermalization is very likely to apply to QCD processes with many final state particles.

**Note:** Eigenvalue spectrum of Dirac operator in QCD is described by RMT at small eigenvalues (at  $T=0$ !)

Relation to theory of RHIC processes?

Most approaches are based on reduced descriptions

$$\text{e.g. } f_1(p, t) = \int dp_2 \dots dp_N \tilde{\phi}(\vec{p}, t)^* \tilde{\phi}(\vec{p}, t)$$

$$\text{or } \rho(\vec{x}, t) = \int dx_2 \dots dx_N \phi(\vec{x}, t)^\dagger \phi(\vec{x}, t)$$

$$\text{or } f_1(p, x, t) = \int dy dx_2 \dots dx_N e^{ipy} \phi^\dagger(\vec{x} + \frac{1}{2}y, t) \phi(\vec{x} - \frac{1}{2}y, t)$$

Coarse graining already involves "entropy generation".

To a certain extent:

Equilibration is in the eye of the beholder!

Important question:

Is the physics of interest - e.g. the Equation of State - sensitive to the discarded details?

⇒ Does EOS of QCD have an effective quasiparticle representation in the accessible  $T$ -range?

Is chemical equilibration driven by  $2 \rightarrow 2$  processes?

If thermal equilibration is driven by  $2 \rightarrow n > 2$  processes, what about  $n \rightarrow m$  for  $n > 2$ ?

Is there a diluteness parameter that justifies the reduced description?

## CONCLUSION

LOTS OF REASONS FOR THEORISTS  
TO THINK ABOUT THE JUSTIFICATION  
OF THEIR METHODS + APPROACHES

LOTS OF REASONS FOR PHENOMENOLOGISTS  
AND EXPERIMENTERS TO BE CAUTIOUS  
AND CAREFUL IN THEIR STATEMENTS

